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6. AUTHOR(S)

Boris Rozovsky  
Alexander Tartakovsky

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

Center for Applied Mathematical Sciences  
University of Southern California  
1042 Downey Way, DRB-312,  
Los Angeles, CA 90089-1113  
Principal Investigator: Boris Rozovskii

8. PERFORMING ORGANIZATION  
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13. ABSTRACT (Maximum 200 words)

In the research during the reporting period we focused on the following areas:

1. Nonlinear filtering for acutely maneuvering targets
2. Development of banks of interacting Bayesian spatial-temporal matched filters for track-before-detect (TBD) based on nonlinear filtering techniques
3. Development of adaptive spatial-temporal filters for clutter rejection and electronic scene stabilization
4. Design of multi-hypothesis sequential tests for multi-sensor distributed systems with fusion of local decisions
5. Wiener chaos expansion for nonlinear systems such with applications to filtering
6. Inverse problems for stochastic PDE.

In addition, we have made substantial progress in the implementation of the developed algorithms. The Adaptive Spatial-Temporal Method for Clutter Rejection and Scene Stabilization and Switching Multiple Model Based TBD Algorithms were transferred to the SPAWAR Systems Center, San Diego, CA (POC: Dr. John Barnett) and Raytheon, El Segundo, CA (EO Signal Processing IPT, POC: Dr. Paul Singer). The algorithms are being inserted into existing test-beds and evaluated for surveillance applications such as cruise missile defense.



14. SUBJECT TERMS

Key words: Interacting Bayesian matched filters, multiple switching models, data fusion, imaging sensors, distributed systems, multihypothesis sequential tests, nonlinear filtering, target detection, target tracking.

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**AUTHORS: Boris Rozovskii and Alexander Tartakovsky**

**PERFORMING ORGANIZATION:**

Center for Applied Mathematical Sciences  
University of Southern California  
1042 West 36th Place, DRB-308  
Los Angeles, CA 90089-1113

**Principal Investigator: Boris Rozovskii**

**SPONSORING/MONITORING AGENCY:**

U. S. Army Research Office  
4300 South Miami Blvd  
Research Triangle Park, NC 27709-2211

**POC/Program Director: Dr. Robert Launer**

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## 1. LIST OF PUBLICATIONS

### 1.1. List of Edited Volumes.

- (1) R. Carmona and B.L. Rozovskii (editors). *Stochastic Partial Differential Equations: Six Perspectives*, volume 64 of *Mathematical Surveys and Monographs Series*. AMS, Providence, RI, 1998.
- (2) Yu. M. Kabanov, B. L. Rozovskii, and A. N. Shiryaev, editors, *Statistics and Control of Stochastic Processes: In honour of R. Sh. Liptser*, pages 139–155. World Scientific, Singapore, 1998.

### 1.2. List of Papers Published in Peer-Review Journals.

- (1) J. Cvitanic, R. Liptser, and B. Rozovskii. "Tracking Volatility", In *Proc. 39th IEEE Conf. on Decision and Control, Sydney, 2000*
- (2) V. Dragalin, A.G. Tartakovsky, and V. Veeravalli, "Multihypothesis Sequential Probability Ratio Tests, Part 1: Asymptotic Optimality," *IEEE Transactions on Information Theory*, vol. 45, No. 7, pp. 2448-2461, 1999.
- (3) V. Dragalin, A.G. Tartakovsky, and V. Veeravalli, "Multihypothesis sequential probability ratio tests, part 2: accurate asymptotic expansions for the expected sample size," *IEEE Transactions on Information Theory*, vol. 46, No. 4, pp. 1366–1383, 2000.
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- (7) S. Lototsky and B. L. Rozovskii. Parameter Estimation for Stochastic Evolution Equations with Non-commuting Operators. In V. Korolyuk, N. Portenko, and H. Syta, editors, *Skorokhod's Ideas in Probability Theory*. Institute of Mathematics of the National Academy of Sciences of Ukraine, Kiev, Ukraine, 2000.
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- (9) K. Ito and B. L. Rozovskii, "Approximation of the Kushner equation for nonlinear filtering," *SIAM J. Control Optim.*, 38(3):893–915, 2000.
- (10) R. Mikulevicius and B. L. Rozovskii. Normalized stochastic integrals in topological vector spaces. In *Seminaire de Probabilites, XXXII*, volume 1686 of *LNM*, pages 137–165. Springer, Berlin, 1998.

- (11) R. Mikulevicius and B. L. Rozovskii. Parabolic stochastic PDE's and Wiener chaos. *SIAM J. Math. Anal.*, 29(2):452–480, 1998.
- (12) R. Mikulevicius and B. L. Rozovskii, “Fourier-Hermit expansion for nonlinear filtering,” *Theory Probab. Appl.*, vol. 44, no. 3, pp. 606–612, 2000.
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- (14) R. Mikulevicius and B. L. Rozovskii. Stochastic Navier-Stokes Equations. Propagation of Chaos and Statistical Moments. In J. L. Menaldi, E. Rofman, and A. Sulem, editors, *Optimal Control and Partial Differential Equations: In honour of Alain Bensoussan*, pages 258–267. IOS Press, Amsterdam, 2001.
- (15) R. Mikulevicius and B.L. Rozovskii. Martingale problems for nonlinear SPDE's. In R. Carmona and B. L. Rozovskii, editors, *Stochastic Partial Differential Equations. Six Perspectives*, volume 64 of *Mathematical Surveys and Monographs Series*, pages 243–325. AMS, Providence, RI, 1998.
- (16) A.G. Tartakovsky, “Asymptotic Optimality of Certain Multihypothesis Sequential Tests: Non-i.i.d. Case,” *Statistical Inference for Stochastic Processes*, vol. 1, No. 3, pp. 265–295, 1998.
- (17) A.G. Tartakovsky, “Asymptotically Optimal Sequential Tests for Nonhomogeneous Processes,” *Sequential Analysis*, vol. 17, No. 1, pp. 33–61, 1998.

### 1.3. List of Conference Papers.

- (1) K. Ito and B. Rozovskii. Splitting-up discretization for Kushner's equation of nonlinear filtering. In *Proc. 36th IEEE Conference on Decision and Control, San Diego, CA, 1998*.
- (2) B. L. Rozovskii and S. Kligys. Matched filters and hidden Markov models with distributed observation. In *Proceedings of the Fourth Annual U.S. Army Conference on Applied Statistics, 21-23 October 1998, Army Research Laboratory, Aberdeen Proving Ground, MD 21005-5067*, pages 183–192, 1998.
- (3) B. L. Rozovskii and S. Kligys. State estimation in hidden Markov models with distributed observation. In Tornambe et al., editor, *Theory and Practice of Control Systems, Proceedings of the 6th IEEE Mediterranean Conference*, volume A, 1999.
- (4) A. Tartakovsky and V. Veeravalli, “Asymptotic Analysis of Bayesian Quickest Change Detection Procedures,” *Proceedings of the IEEE International Symposium on Information Theory (ISIT)*, Lausanne, Switzerland, 30 June–5 July, 2002.
- (5) A. Tartakovsky, “An Efficient Adaptive Sequential Procedure for Detecting Targets,” *Proceedings of the IEEE Aerospace Conference*, Big Sky, Montana, 9-16 March 2002.
- (6) R. Blažek, H. Kim, B. Rozovskii, and A. Tartakovsky, “A Novel Approach to Detection of “Denial-of-Service” Attacks Via Adaptive Sequential and Batch-Sequential Change-Point Detection Methods,” *IEEE Systems, Man, and Cybernetics Information Assurance Workshop*, West Point, NY, 5-6 June 2001.
- (7) B.L. Rozovskii, A. Petrov, and R.B. Blažek, “Interactive banks of Bayesian matched filters,” *SPIE Proceedings: Signal and Data Processing of Small Targets*, Vol. 4048, (O.E. Drummond, Ed.), Orlando, FL, 2000.

- (8) B. Rozovskii, A.G. Tartakovsky, and G. Yaralov, "An Adaptive Bayesian Approach to Fusion of Imaging and Kinematic Data," *Proceedings of the 2nd International Conference on Information Fusion (Fusion'99)*, vol. 2, pp. 1029-1036, Sunnyvale, CA, 6-8 July, 1999.
- (9) A. Tartakovsky and X.R. Li, "Invariant Sequential Detection and Recognition of Targets in Distributed Systems," *Proceedings of the 4rd International Conference on Information Fusion*, Montreal, Canada, 7-10 August 2001, Volume II, pp. WeA3-11-WeA3-18.
- (10) A. Tartakovsky, "Quickest Detection of Targets in Multiple-Resolution-Element Systems: Sequential Detection Versus Non-Sequential Detection," *Proceedings of the Workshop on Estimation, Tracking, and Fusion: A Tribute to Yaakov Bar-Shalom*, pp. 326-350, Monterey, CA, 17 May, 2001.
- (11) A. Tartakovsky and X.-R. Li, "Sequential testing of multiple hypotheses in distributed systems," *The 3rd International Conference on Information Fusion (Fusion'00)*, Paris, France, July 10-13, 2000.
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#### 1.4. List of Papers Presented at Meetings.

- (1) R. Blazek, H. Kim, and B. L. Rozovskii. A New Adaptive Batch and Sequential Methods for Rapid Detection of Network Traffic Changes with Emphasis on Detection of "Denial-of-Service" Attacks. In *Proceedings of the 53rd Session of the International Statistical Institute, Seoul*, 2001.
- (2) B. Rozovskii, A. Tartakovsky and R. Blažek, "Challenges in Spatial-Temporal Method for Electronic Scene Stabilization and Clutter Removal, and Nonlinear Filtering for Track Before Detect", *4rd ONR/GTRI Workshop on Target Tracking and Sensor Fusion*, Monterey, May 15-16, 2001.
- (3) R. Blažek, A. Petrov, B. Rozovskii, and A. Tartakovsky, "Adaptive Spatial-Temporal Method for Clutter Rejection and Switching Multiple Model Based Target Tracking Algorithms", *3rd ONR/GTRI Workshop on Target Tracking and Sensor Fusion*, Atlanta, May 17-18, 2000.
- (4) A.G. Tartakovsky, "Asymptotic optimality of certain change-point detection procedures: non-i.i.d. case," *5th World Congress of the Bernoulli Society for Mathematical Statistics and Probability*, Guanajuato, Mexico, May 12-20, 2000.

- (5) A. Tartakovsky, "An Optimal Solution to a Multi-Population Quickest Detection Problem," *The 6th ASA/IMS Spring Research Conference on Statistics in Industry and Technology*, Minneapolis, Minnesota, 2-4 June, 1999. IMS Bulletin, vol. 28, No. 2, p. 110, 1999.
- (6) A.G. Tartakovsky, "Asymptotic Properties of M-SPRT and Adaptive Sequential Test in Multi-hypothesis Problems With an Indifference Zone," *'98 Joint Statistical Meetings*, Dallas, Texas. IMS Bulletin, vol. 27, No. 3, p. 167, 1998.
- (7) A.G. Tartakovsky, "Multihypothesis Sequential Tests With Application to Target Identification in Multi-Resolution Systems: Asymptotic Optimality," *IMS/SPES Spring Research Conference*, Santa Fe, New Mexico, 1998. IMS Bulletin, vol. 27, No. 2, pp. 92-93, 1998.

### 1.5. Submitted Manuscripts.

- (1) A.G. Tartakovsky, X.R. Li, and G. Yaralov, "Sequential Detection of Targets in Distributed Systems," *IEEE Transactions on Information Theory* (submitted 2001).
- (2) A.G. Tartakovsky, "Extended Asymptotic Optimality of Certain Change-Point Detection Procedures," *The Annals of Statistics* (submitted 2000).

### 1.6. List of Technical Reports and Preprints.

- (1) S. Kligys, B. Rozovsky and A. Tartakovsky, "Detection Algorithms and Track Before Detect Architecture Based on Nonlinear Filtering for Infrared Search and Track Systems," *Technical Report CAMS-98.9.1*, Center for Applied Mathematical Sciences, University of Southern California, 1998.
- (2) C. Rao, B. Rozovsky, and A. Tartakovsky, "Domain Pursuit Method for Tracking Ballistic Targets," *Technical Report CAMS-98.9.2*, Center for Applied Mathematical Sciences, University of Southern California, 1998.

## 2. SCIENTIFIC PERSONNEL SUPPORTED BY THIS PROJECT

- (1) M. Kinarivala (Research Assistant at CAMS, USC) – M.S. student, Department of Electrical Engineering-Systems, USC
- (2) R. Mikulevicius (Professor, Department of Mathematics, USC)
- (3) B. Rozovskii (Professor, Department of Mathematics, CAMS Director, USC) – PI of the project

## 3. IMPACT/APPLICATIONS

For performance evaluation and tuning, we used the real IR background data obtained from SPAWAR Systems Center, San Diego, CA (staring shipboardIRST). It turns out that the developed algorithms are able to detect very low SNR targets – down to  $-9\text{dB}$  (see [22, 31, 36] for more details). It is expected that the developed algorithms will be successfully used in EO/IRST systems in 6.2 programs for the detection and tracking of low-SNR targets. In particular, currently the developed algorithms are being evaluated for surveillance applications by Raytheon, El Segundo, CA and SPAWAR Systems Center, San Diego, CA (see Section 4).



#### 4. TECHNOLOGY TRANSFER

The Adaptive Spatial-Temporal Method for Clutter Rejection and Scene Stabilization and Switching Multiple Model Based TBD Algorithms were transferred to the SPAWAR Systems Center, San Diego, CA (POC: Dr. John Barnett) and Raytheon, El Segundo, CA (EO Signal Processing IPT, POC: Dr. Paul Singer). The algorithms are being inserted into existing testbeds and evaluated for surveillance applications such as cruise missile defense.

#### 5. SCIENTIFIC PROGRESS AND ACCOMPLISHMENTS

During the reporting period the following work was completed:

- (1) Development of a spectral approach to nonlinear filtering based on Wiener Chaos expansions
- (2) Development of the theory and applications of inverse problems for stochastic PDE's.
- (3) Development of novel methodology for clutter rejection and electronic scene stabilization.
- (4) Development of the adaptive spatial-temporal algorithms for clutter rejection and electronic scene stabilization based on different time-splitting approximation schemes that use different spatial bases (Fourier, wavelets, etc.).
- (5) Performance evaluation of these algorithms and their comparison with the best spatial-only techniques.
- (6) Development of optimal banks of interacting Bayesian matched filters for TBD.
- (7) Development and implementation of different fast numerical approximations of this algorithm, including fast Gaussian-mixture approximations.
- (8) Performance evaluation of BIBMF and its comparison with the IMM approach, banks of 3D matched filters, and Viterbi-type algorithms.
- (9) Development of sequential multihypothesis tests with data fusion in multi-sensor distributed systems.
- (10) Performance evaluation of fusion local sequential decisions in distributed systems.

**5.1. Wiener Chaos Expansion as a Numerical Algorithm.** We derived and investigated deterministic equations for the Hermite-Fourier coefficients of the Wiener chaos expansion of a solution to Duncan-Mortensen-Zakai equation of nonlinear filtering. These result was used for development of numerical spectral separation schemes for solving those equations. We demonstrated that spectral separating scheme complemented by the domain pursuit method provides quite satisfactory results in higher dimensions ( $d = 4 - 6$ ).

**5.2. Nonlinear Filtering for Doubly Stochastic Models with Jumps.** We studied detection and tracking of maneuvering low intensity (dim) targets in image sequences (e.g. infrared imaging). Traditionally, the uncertainty in the trajectory of a maneuvering non-cooperative target is modelled by an additive state noise. This type of modelling is clearly insufficient for agile acutely maneuvering targets. Thus, to account for sharp maneuvers, we allowed doubly stochastic models for the state process. More specifically, we studied state models given by a linear Ito diffusion process  $X_t$  with coefficients depending on a Markov jump process  $\theta_t$ . The latter process models transitions between the base states (possible maneuvers). Models of this type are often referred to in the literature as switching

multiple models, Markov modulated models or affine models. Their main advantages are in flexibility and computational simplicity. The latter stems from the linearity of the state process for fixed value of the switching process  $\theta_t$ .

**5.3. Banks of Interacting Bayesian Matched Filters.** The proliferation of imaging sensors (such as IR, SAR, HRRR, etc.) has been an important trend in the development of military tracking and detection systems for over a decade. This trend put forth a number of new challenging problems in signal processing.

The most accepted approach to tracking based on imaging data is the 3D matched filter proposed by Reed et al. [30]. This technique is known to produce excellent results provided the target is moving at a known speed in a designated direction. This limitation could be offset, at least partially, by the use of a bank of assumed velocity filters. Still, applications of 3D matched filtering are limited to a somewhat narrow set of patterns of target dynamics. In particular, the 3D match filter is poorly equipped for handling target kinematics with rapid switches between multiple models. It does not have a built-in mechanism for incorporating statistically formalized prior knowledge about the target into data association.

In [23, 24, 31] we demonstrated that the 3D matched filter can be cast into a general framework of optimal spatial-temporal Bayesian filtering. This allows us to extend the matched filtering algorithm to a wide class of models of target dynamics, including switching multiple models (SMM). In the reporting period, this idea has been implemented for the development of Banks of Interacting Bayesian Matched Filters (BIBMF).

BIBMF turns out to be a theoretically optimal algorithm even for nonlinear models for observations which are typical for TBD in imaging sensors. For nonlinear models considered in the research a standard IMM algorithm proposed by Blom and Bar Shalom [7, 10] cannot be applied at all. We, however, showed that our method is more efficient even in a linear case where IMM proved to work well. In addition, we compared the developed structure with two other algorithms: banks of 3D matched filters and the Viterbi type batch algorithm. The latter one is also our proprietary algorithm that was probably developed for these applications for the first time.

BIBMF along with the other above mentioned algorithms were tested on an important and difficult problem of tracking-before-detection of maneuvering targets. Real IR background data (courtesy of SPAWAR Systems Center, San Diego, CA) were used in this test. Robust and accurate performance was demonstrated for very low SNR targets (up to  $-6.6$  dB). The results revealed that BIBMF substantially outperformed all other algorithms.

Below, we present the test results of the developed BIBMF algorithm for tracking-before-detection of agile targets in IR cluttered 2D images. The results show that the BIBMF algorithm is able to handle acutely maneuvering targets with very low SNR.

For the simulation study, a maneuvering target of the size  $3 \times 3$  pixels was artificially superimposed on the imagery. The impulse function  $\mathbf{h}(\mathbf{x})$  is constant (with amplitude  $S$ ) over the target image ( $3 \times 3$  pixels) and zero elsewhere on the sensor array. The residual background (residual clutter plus sensor noise),  $V_k^i$ , is modeled as a space-time white Gaussian noise,  $V_k^i \sim \mathcal{N}(0, \sigma_0^2)$ . Two sets of experiments



were conducted with the residual (after preprocessing) single-pixel SNR fixed at 0 dB and -3 dB levels. SNR is defined by  $SNR = 20 \log_{10}(S/\sigma_0)$ , where  $\sigma_0$  is the standard deviation of the residual background noise.

In every experiment, the initial position of the target was uniformly distributed and chosen randomly. The initial state of the switching process was also uniformly distributed among 9 possible models of kinematics. The basic state models are shown in Figure 1. Five of them represent constant speed linear motion in assumed direction, and another four represent constant rate  $90^\circ$  turns. The switching probabilities between different basic states were chosen as follows. If the target performs a linear motion, then the probability that it preserves the same direction (about 0.8) is larger than the probabilities of switching to different modes. In contrast, if the target is currently performing a turn maneuver, we assign a dominating probability (about 0.9) to switching to the linear motion which is tangential to the target curve. The rest of probability is distributed uniformly among those turn maneuvers which the target is capable of making after completing the current one (see Figure 1). The model described above incorporates quite complicated trajectories and allows for frequent maneuvering.

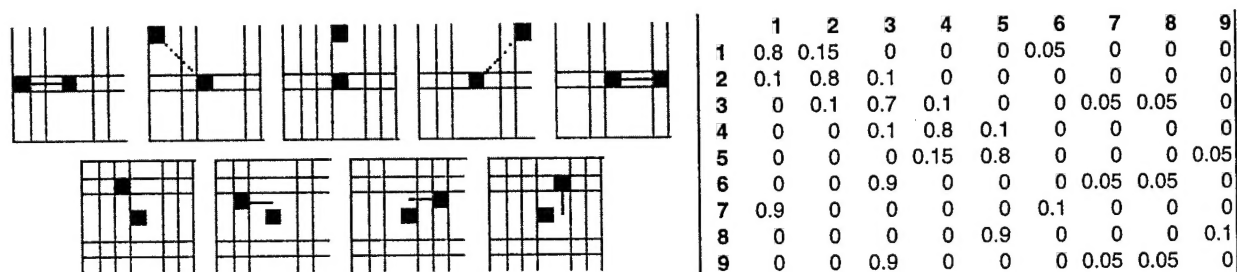


FIGURE 1. Possible target movements (left) and transition probabilities between multiple models (right)

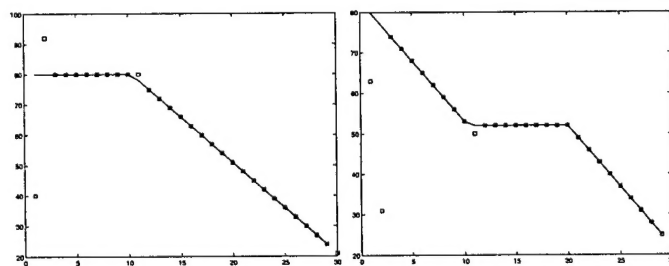


FIGURE 2. Actual trajectory (solid line) and maximum posterior density estimates (squares),  $x$  coordinate versus time (left),  $y$  coordinate versus time (right), SNR = -3dB

Figure 2 shows the results of tracking with the use of the maximum posterior estimator for quite low SNR. It is seen that the true trajectory was estimated very accurately.

An important parameter of a TBD algorithm is the number of frames necessary for an "accurate" estimation of target location. The average delays in target detection after its first appearance and after a turn maneuver were estimated by using the Monte Carlo experiment. The results of this experiment are as follows.

- SNR=0 dB: after the first appearance—0.9 frames, after a turn maneuver—0.2 frames
- SNR= -3 dB: after the first appearance—5.0 frames, after a turn maneuver—1.7 frames

**5.4. Adaptive Spatial-Temporal Method for Clutter Rejection and Electronic Scene Stabilization.** We start with the discussion of the performance indices. Let  $S_n(\mathbf{r}_{ij})$  and  $\tilde{S}_n(\mathbf{r}_{ij})$  be the original signal from the target and the signal after clutter rejection, respectively. Here  $\mathbf{r}_{ij} = (x_i, y_j)$  is the pixel with coordinates  $(x_i, y_j)$  on the plane. Introduce the following indices:

$$I_1 = \frac{\sum_{i,j} S_n^2(\mathbf{r}_{ij})}{\sum_{i,j} \tilde{S}_n^2(\mathbf{r}_{ij})}, \quad I_2 = \frac{\sum_{i,j} S_n^2(\mathbf{r}_{ij}) \tilde{S}_n^2(\mathbf{r}_{ij})}{\sum_{i,j} S_n^2(\mathbf{r}_{ij}) \cdot \sum_{i,j} \tilde{S}_n^2(\mathbf{r}_{ij})}.$$

From the point of view of correct signal reconstruction/preservation, a good algorithm should provide both  $I_1$  and  $I_2$  close to 1. If this is the case, then a good algorithm, from the point of view of clutter rejection, should maximize the value of

$$(1) \quad G = 10 \log \frac{\sigma_{\text{in}}^2}{\sigma_{\text{out}}^2},$$

where  $\sigma_{\text{in}}^2$  is the variance of the input frame  $\mathbf{Y}_n$  and  $\sigma_{\text{out}}^2$  is the variance of the output frame  $\tilde{\mathbf{Y}}_n$ . Indeed, if the signal is preserved, then the maximization of  $G$  is equivalent to the maximization of the relative Signal-to-Noise-Plus-Clutter Ratio

$$10 \log \frac{\text{SNCR}_{\text{in}}}{\text{SNCR}_{\text{out}}}$$

where

$$\text{SNCR}_{\text{in}} = \sqrt{\frac{\sum_{i,j} S_n^2(\mathbf{r}_{ij})}{\sigma_{\text{in}}^2}}, \quad \text{SNCR}_{\text{out}} = \sqrt{\frac{\sum_{i,j} \tilde{S}_n^2(\mathbf{r}_{ij})}{\sigma_{\text{out}}^2}}.$$

Thus, we will use the index  $G$  defined in (1) as the measure of the quality of the clutter rejection: the bigger  $G$ , the better the algorithm.

In simulations we used a subset  $\mathcal{H}$  of the Haar wavelet basis to approximate the clutter function  $b_n(\mathbf{r})$  in the sequence of frames  $\mathbf{Y}_n(\mathbf{r}) = b_n(\mathbf{r}) + \xi_n(\mathbf{r})$ , where  $\xi_n(\mathbf{r})$  is sensor noise.

In the observations  $\mathbf{Y}_k, k = 1, 2, \dots, n$ , the two-dimensional parallel jitter  $\{\delta_k = (\delta_{x,k}, \delta_{y,k})\}$  was modeled by pairs of independent random variables uniformly distributed over the set  $\{0, \pm 1, \dots, \pm \delta_{\text{max}}\}$ . The observations  $\mathbf{Y}_1, \dots, \mathbf{Y}_n$  were generated by applying  $n$  independent replicas of the jitter to the coordinates of the function  $b$ , and then, subsequently adding white Gaussian noise (with mean 0 and variance  $\sigma^2$ ) to each component of a discretized version of the function  $b(\mathbf{r} + \delta_k)$ .

Figure 3 illustrates the performance of the developed rejection filter for a particular case. In Figure 3, the picture on the left-hand side shows a typical input (cluttered and noisy) frame with the noise variance  $\sigma^2 = 10$ , clutter dynamic range (CDR) 10 – 100, and jitter  $\delta \in [-2, +2]$  pixels. The picture on the right side shows the result of clutter rejection (the residuals at the output of the filter) with temporal window size  $T = 20$  frames. It can be seen that clutter is completely removed and the residuals look like noise. For comparison, Figure 4 illustrates the result of spatial-only (in-frame) processing based on the nonparametric method developed in CAMS previously [25]. This spatial clutter rejection technique is based on nonparametric regression algorithms, namely, on kernel smoothing methods. This technique proved to be highly efficient for a variety of ‘difficult’ cluttered scenes, in particular for the IR LAPTEX field test data (see [25] for more details). In Figure 4, the picture on the left-hand side depicts the

estimate of clutter, while the picture on the right-hand side shows the residuals at the output of the spatial filter. One can see that clutter is removed only partially. The pieces of residual clutter can be seen even by the naked eye. The advantage of the developed temporal-spatial filter over the spatial filter is obvious when comparing the right-hand side pictures in Figure 3 and Figure 4.

The data in Table 1 summarize the performance of the rejection filter in terms of the gain  $G$  defined in (1), and also, in terms of other important characteristics: the dynamic range (maximum and minimum values  $\max_{i,j} Y_n(r_{ij})$  and  $\min_{i,j} Y_n(r_{ij})$ ), mean value  $\bar{Y}_n = (N_1 N_2)^{-1} \sum_{i,j} Y_n(r_{ij})$ , and variance  $\sigma_Y^2 = (N_1 N_2)^{-1} \sum_{i,j} [Y_n(r_{ij}) - \bar{Y}_n]^2$ . For the spatial-temporal filter, the residuals have zero mean value, the dynamic range is much less than in the input frame, and the variance is close to the variance of the noise ( $\sigma_Y^2 = 10.84$  versus  $\sigma^2 = 10$ ). These numbers show that clutter is suppressed down below the noise level. This allows us to arrive at the conclusion that the developed algorithm is highly efficient: it completely removes high-intensive clutter in the presence of substantially large jitter. Also, the data in Table 1 allow us to compare the nonparametric spatial filter with the developed spatial-temporal filter at the qualitative level. It can be seen that the dynamic range of the output frame of the spatial-temporal filter is 3 times smaller than that of the output frame of the spatial filter. The variance of the output frame of the spatial-temporal filter is over 10 times (10.3 dB) smaller than that of the output frame of the spatial filter.

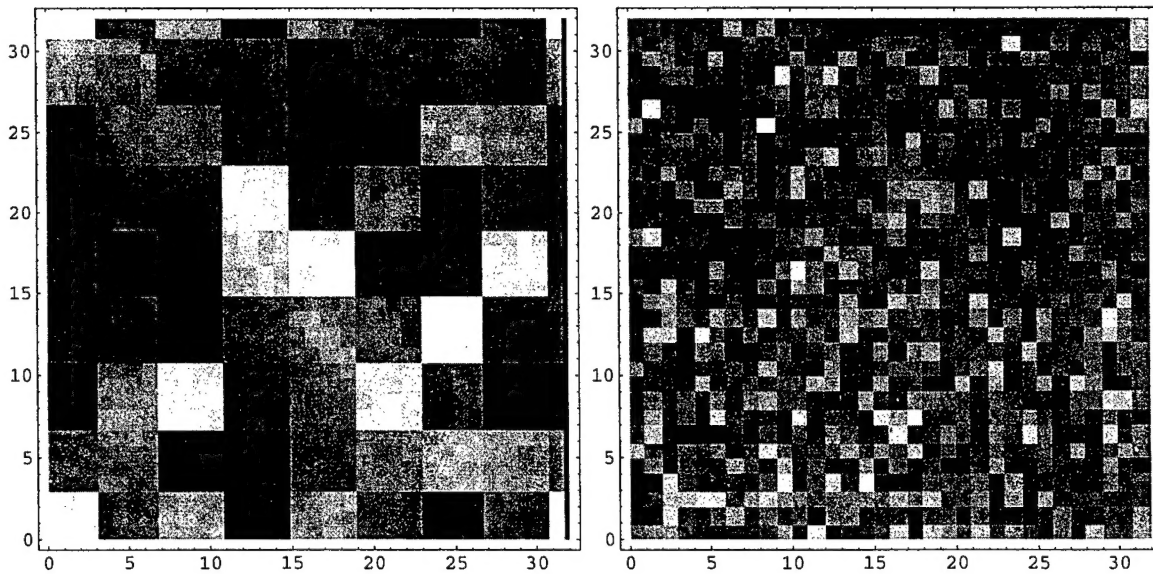


FIGURE 3. Clutter Rejection: Spatial-Temporal Filter with the Haar Basis ( $\sigma^2 = 10$ ,  $\delta = \pm 2$ , CDR = 10 – 100,  $T = 20$ )

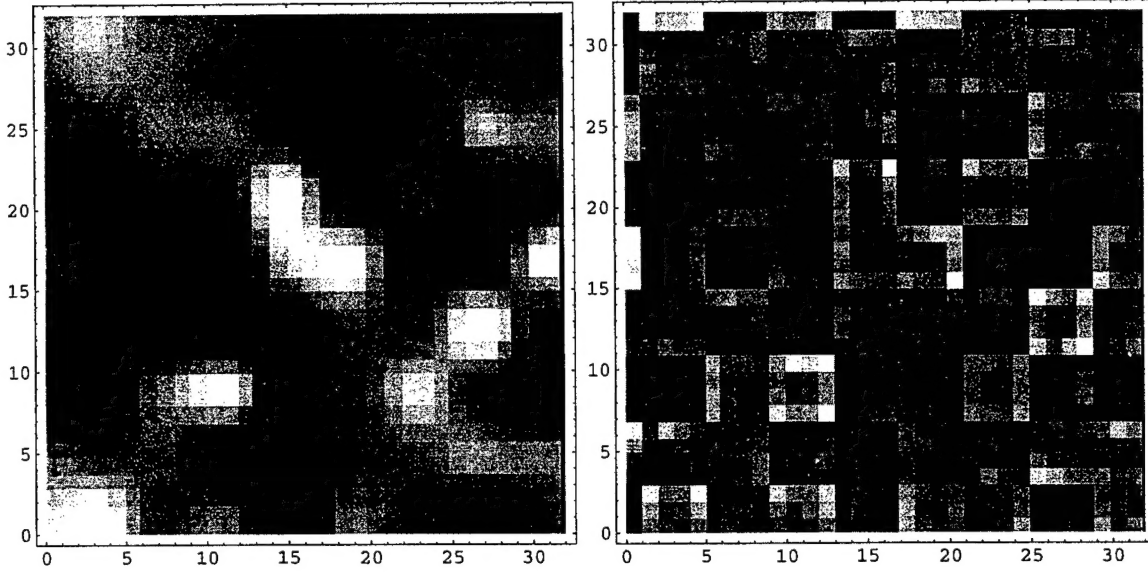


FIGURE 4. Clutter Rejection: Spatial Nonparametric Filter ( $\sigma^2 = 10$ , CDR = 10 – 100)

TABLE 1. Performance of Clutter Rejection Algorithms ( $\sigma^2 = 10$ ,  $\delta = \pm 2$ )

	<i>Minimum</i>	<i>Maximum</i>	<i>Mean</i>	<i>Variance</i>	<i>Gain</i>
<b>Input</b>	3.04	105.25	56.10	482.86	
<b>Output (spatial-temporal), <math>T = 20</math></b>	-10.24	9.44	-0.011	10.84	16.5 (dB)
<b>Output (spatial)</b>	-36.34	34.60	-0.004	114.99	6.2 (dB)

**Discussion of the results.** The development of efficient IR clutter rejection algorithms is of critical importance for modern IRST systems. LOS stabilization jitter, which results in translational, rotational, and parallax distortions in registered images, does not allow for efficient temporal filtering of frames and clutter rejection. This is probably one of the major reasons why current IR scanning and staring array sensors employ primarily spatial, rather than spatial-temporal, processing to accomplish clutter rejection.

We proposed a novel approach to spatial-temporal clutter rejection and scene stabilization. This approach includes a jitter estimation and compensation algorithm as a non-separable part. The proposed clutter rejection method does not use any assumptions on statistical models of clutter, which are usually unreliable and lead to non-robust algorithms. All we need for efficient temporal processing is the condition that clutter does not change substantially on a certain time interval. As a result, the developed rejection filter is highly robust and can handle any spatial variations of clutter.

Based on the results of simulations, we can conclude that the developed algorithm is highly efficient: it completely removes high-intensive clutter in the presence of substantial jitter. Also, the spatial-temporal filter gives a tremendous gain compared to the best existing spatial techniques.

### 5.5. Sequential Testing of Multiple Hypotheses in Multi-Sensor Distributed Systems.

Most of the research in fusion of data from multiple sensors was done in a non-sequential setting (see [4, 11, 13, 16] and many others) where the differences among sensor decision times, and their differences from the fusion time are ignored. In many practical systems, however, sensor decisions are made in a sequential manner at random times, depending on the data that are received sequentially by the sensors. In some other cases, different sensor decisions are made at different, albeit fixed, times when the sensors utilize decision rules with fixed (but different) sample sizes. It is, therefore, important to consider sensor decisions and their fusion in a sequential setting where either the fusion rule or the sensor rules are sequential in nature.

In this research, we study the problem of fusing local decisions made sequentially by multiple sensors. We consider a sequential version of the distributed decision problem that has predetermined sensor decision rules. It is assumed that each sensor sequentially tests  $M$  hypotheses, and then, the  $M$ -ary local decisions are transmitted to a fusion sensor, one-by-one, in the order they are made. A fusion center combines these local decisions to further test hypotheses either sequentially or non-sequentially. As a result, the performance is enhanced. We do not assume that observations are i.i.d. In contrast, it is assumed that the observations can be highly correlated and non-stationary, which is important for many applications. The proposed  $M$ -ary sequential test turns out to be asymptotically optimal for very general statistical models when the probabilities of errors are small. In general, we do not also assume that the local decisions that are transmitted to the fusion center are independent. However, in this general case, it is difficult to design final fusion rules to meet constraints on the error probabilities and/or to compute performance. This design is performed in the case where the local decisions are independent (not necessarily identically distributed).

Performance analysis shows that the final decision (after fusion of local decisions) can be made substantially more reliable even for a small number of sensors (3-5).

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